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A CONSERVATION LAW WITH MULTIPLY DISCONTINUOUS FLUX MODELLING A FLOTATION COLUMN

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Abstract. Flotation is a unit operation extensively used in the recovery of valuable minerals in mineral processing and related applications. Essential insight to the hydrodynamics of a flotation column can be obtained by studying just two phases: gas and fluid. To this end, the approach based on the drift-flux theory, proposed in similar form by several authors, is reformulated as a one-dimensional non-linear conservation law with a multiply discontinuous flux. The unknown is the gas volume fraction as a function of height and time, and the flux function depends discontinuously on spatial position due to several feed inlets. The resulting model is similar, but not equivalent, to previously studied clarifier-thickener models for solid-liquid separation and therefore adds a new real-world application to the field of conservation laws with discontinuous flux. Steady-state solutions are studied in detail, including their construction by applying an appropriate entropy condition across each flux discontinuity. This analysis leads to operating charts and tables collecting all possible steady states along with some necessary conditions for their feasibility in each case. Numerical experiments show that the transient model recovers the steady states, depending on the feed rates of the different inlets.

1. Introduction.

1.1. Scope.

Flotation is a unit operation that is extensively used in the recovery of valuable minerals and coals in mineral processing but also in many other applications in environmental and chemical engineering [12, 21, 33, 36, 42]. It is a physico-chemical separation process that utilizes the difference in surface properties of the valuable hydrophobic minerals and the unwanted hydrophilic gangue material. The theory of froth flotation is complex and involves three phases (solids, water, and froth or gas) with many subprocesses [42]. The principle of the conventional flotation process is roughly as follows. Gas is introduced close to the bottom

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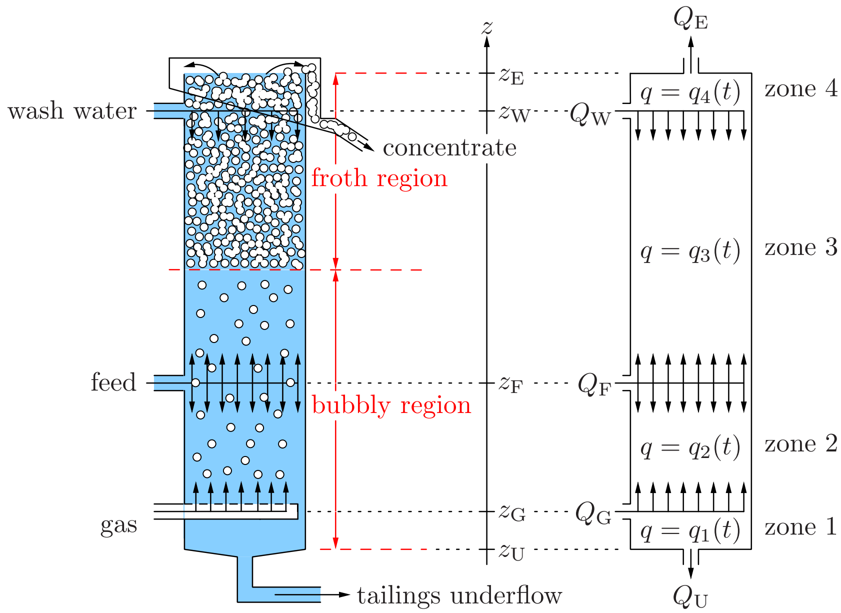

FIGURE 1. Left: Schematic of a typical flotation column (after [21, 38]), including heights of singular sources z_G, z_F and z_W, the underflow level z_U, and the effluent level z_E. Right: Corresponding conceptual model of the flotation column used in this work, indicating the volumetric feed flows Q_G, Q_F and Q_W, the underflow rate Q_U, the effluent rate Q_E, and the spatially piecewise constant bulk velocity $q = q(z, t)$.

of a flotation column (see Figure 1), and the bubbles generated then rise upward throughout the pulp that contains hydrophobic and hydrophilic solid particles. The hydrophobic particles in the pulp attach to the bubbles. Since the overall density of the bubble-particle aggregates is less than that of the medium, the aggregates then float to the top of the column, where the desired product, the foam or froth carrying the valuable material (the concentrate in mining) is removed, usually through a launder. Additionally, close to the top wash water is injected to assist with the rejection of entrained impurities [39] and to increase the froth stability and improve recovery [21, 31]. Once the hydrophobic particles have attached to the air bubbles, flotation can be considered as a separation between relatively large low-density entities, called air bubbles, and a suspension of liquid and gangue. Consequently, flotation can be described as a gas-liquid separation process by buoyancy analogous to the solid-liquid separation by gravity sedimentation in clarifier-thickeners [8, 10, 15].

Well-established spatially one-dimensional models of clarifier-thickeners can be formulated as a scalar conservation law for the local solids concentration as a function of depth and time, where the flux is discontinuous as a function of spatial position due to upward- and downward-directed bulk flows, transitions to overflow and underflow transport, and a singular source term marking the feed [8, 10, 15]. Clarifier-thickener models have motivated in part the mathematical research on conservation laws with discontinuous flux [3, 4, 6, 10, 14–20].