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Matching one-loop divergences in 7D Einstein and 6D Conformal Gravities

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Abstract

Within the context of AdS/CFT correspondence, we first compute the one-loop infrared (IR) divergences of 7D Einstein gravity in a certain Poincaré-Einstein background metric. Then, we compute the one-loop ultraviolet (UV) divergences of 6D conformal gravity on the conformal boundary. We verify the equality of the above results that stem from the IR–UV connection of the duality dictionary. Key ingredients are heat kernel techniques, factorization of the boundary higher-derivative kinetic operator for the Weyl graviton on the 6D Einstein metric, and Wentzel–Kramers–Brillouin (WKB) exactness of the Einstein graviton in the chosen 7D Poincaré-Einstein background. Overall, we elucidate the way in which the 6D results containing the type-A and type-B conformal anomalies for the Weyl graviton are encoded in the 7D "hologram" given by the fluctuation determinant for the Einstein graviton. We finally discuss possible extensions to include higher-spin fields.

KEYWORDS

AdS/CFT correspondence, anomalies in quantum field theory

1 | INTRODUCTION

In the mid-1970s, higher-curvature gravities naturally turned up in response to the nonrenormalizable ultraviolet (UV) divergences of Einstein gravity (Deser & van Nieuwenhuizen 1974; Goroff & Sagnotti 1985; 't Hooft & Veltman 1974). Subsequent attention was paid to quadratic gravities in four dimensions once their perturbative renormalizability was established (Stelle 1977). As of today, quadratic and higher-curvature gravities can be embedded into a fundamental theory such as string or M theory, and the notorious lack of unitarity can be attributed to an artifact of the truncation of the otherwise ghost-free UV completion (Alvarez-Gaume et al. 2016).

Motivated mainly by developments in string theory, the study of higher-derivative gravities in dimensions larger than four also gained a renewed interest. Take, for example, six dimensions: there is only one Gauss-Bonnet term that cannot absorb the dependence on the Riemann or Weyl tensor of the one-loop divergences, and as a result, pure Einstein gravity turns out to be already nonrenormalizable at one loop. The equivalent role of 4D quadratic gravities is played here by six-derivative gravities containing cubic powers of Ricci and Riemann tensors. In particular, 6D Weyl or conformal gravities built up out of the three 6D pointwise Weyl invariants are indeed one-loop power-counting renormalizable.

In the present work, we pay attention to a particular 6D conformal gravity and study the precise structure of its one-loop UV-log divergences and, eventually, the way they fit into the AdS/CFT correspondence. There are at least two features (Beccaria & Tseytlin 2017) that single out this

6D conformal gravity: (a) it vanishes on a Ricci-flat background, and (b) it admits a (2,0) supersymmetric extension. The first one is best known to conformal geometers for this is a crucial property of Branson's Q-curvature, the quantity that arises within AdS/CFT as the volume anomaly of (asymptotically AdS) Poincaré-Einstein metrics. The second property is related to the fact that the very same combination of pointwise Weyl invariants appears in the accumulated b_6 heat coefficient for the free (2,0) tensor multiplet (Bastianelli et al. 2000).

The precise combination of pointwise Weyl invariants that makes up the 6D conformal gravity under consideration is the following

$$S_{CG} = \int d^6x\sqrt{g}\left[Ric\nabla^2 Ric - \frac{3}{10}R\nabla^2 R + 2RiemRic^2 - RRic^2 + \frac{3}{25}R^3\right]. \qquad (1)$$

The one-loop partition function for the corresponding 6D Weyl graviton can be obtained by integrating the quadratic metric fluctuations after fixing the Feynman–de Donder gauge and taking into account the ghost contribution. The major technical difficulty in doing so is posed by the six-order kinetic operator acting on the transverse-traceless metric fluctuations. However, restricting to an Einstein background, the computations are greatly simplified, and we end up with the following quotient of functional determinants of (minimal) second-order differential operators, simple-shifted Lichnerowicz Laplacians,

$$Z^{1-loop}_{Weyl} = \left[\det\left\{\Delta_L^{(1,\perp)} - \frac{R}{3}\right\}\det\left\{\Delta_L^{(0)} - \frac{R}{5}\right\}\right]^{1/2} \times \left[\det\left\{\Delta_L^{(2,\perp\top)} - \frac{R}{3}\right\}\det\left\{\Delta_L^{(2,\perp\top)} - \frac{R}{5}\right\} \det\left\{\Delta_L^{(2,\perp\top)} - \frac{2R}{15}\right\}\right]^{-1/2}. \qquad (2)$$

This factorized form first appeared in the physics literature (Pang 2012), but it was incorrectly claimed to hold up only on symmetric Einstein manifolds. An extension of the factorization to Ricci-flat manifolds was exploited later on (Beccaria & Tseytlin 2017). In greater generality, the factorization of the second metric variation of the (critical) Q-curvature on a generic Einstein manifold was established (Matsumoto 2013) using the Fefferman–Graham ambient metric construction.

The structure of the UV-log divergences of any 6D Weyl invariant action is dictated by the trace (or Weyl or conformal) anomaly (Bastianelli et al. 2000):

$$\mathcal{A}_6 = -aE_6 + c_1 I_1 + c_2 I_2 + c_3 I_3. \qquad (3)$$

The restriction to symmetric Einstein spaces, such as $S^6, S^2 \times S^4, S^3 \times S^3, S^2 \times S^2 \times S^2$, where the Weyl tensor is covariantly constant, grants access to the coefficient of the Pfaffian E_6 but forces a linear relation between the pointwise Weyl invariants $5I_3 = 32I_1 - 8I_2$. As a consequence, there is not enough information to disentangle the four anomaly coefficients from only three independent terms. This restricted approach to the determination of the UV-log divergences of the one-loop effective action for the 6D Weyl graviton was carried out (Pang 2012) by explicit computation of the eigenvalues and degeneracies of the second-order differential operators that enter the functional determinants. The partial information obtained by this procedure was then the following

$$a = \frac{601}{2016}; \quad c_1 + 4c_2 = \frac{5633}{105}; \quad c_3 - \frac{5}{8}c_2 = -\frac{35543}{5040}. \qquad (4)$$

More recently, the restriction to a Ricci-flat, but not conformally flat, background was considered (Beccaria & Tseytlin 2017). Ricci flatness forces two linear relations between the four terms of the anomaly basis, namely, $E_6 = 64I_1 + 32I_2$ and $I_3 = 4I_1 - I_2$, so that there are only two independent terms in the anomaly. The coefficients of these two independent combinations were obtained by explicit evaluation of the accumulated b_6 heat kernel coefficients of all the second-order kinetic operators involved. When combined with the previous partial results from symmetric Einstein manifolds, the first term brings in no new information, but the second one allows the complete determination of c_3 and, in consequence, of c_1 and c_2. As a result, both computations nicely complement each other to produce the full set of central charges

$$a = \frac{601}{2016}; \quad c_1 = \frac{1507}{45}; \quad c_2 = \frac{635}{126}; \quad c_3 = -\frac{1639}{420}. \qquad (5)$$

Let us now turn to the holographic side of this story: somewhat unexpectedly, this 6D conformal gravity computation has a 7D bulk counterpart within AdS/conformal field theory (CFT) correspondence. There is a kinematic relation between the one-loop partition functions of the bulk Einstein graviton and the boundary Weyl graviton (Giombi et al. 2013)

$$\frac{Z^{1-\text{loop},-}_{\text{Einstein}}}{Z^{1-\text{loop},+}_{\text{Einstein}}} = Z^{1-\text{loop}}_{\text{Weyl}}. \qquad (6)$$

The bulk side contemplates the ratio of the functional determinants of the kinetic operator of the bulk field computed with standard and alternate boundary conditions, whereas the boundary side involves the functional determinant of the kinetic operator of the induced field. This kind of *holographic formula* was obtained via a rather cir-