



Generalized Maxwellian exotic Bargmann gravity theory in three spacetime dimensions



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ARTICLE INFO

Article history:

Received 11 April 2020

Received in revised form 21 June 2020

Accepted 22 June 2020

Available online 29 June 2020

Editor: N. Lambert

ABSTRACT

We present a generalization of the so-called Maxwellian extended Bargmann algebra by considering a non-relativistic limit to a generalized Maxwell algebra defined in three spacetime dimensions. The non-relativistic Chern-Simons gravity theory based on this new algebra is also constructed and discussed. We point out that the extended Bargmann and its Maxwellian generalization are particular sub-cases of the generalized Maxwellian extended Bargmann gravity introduced here. The extension of our results using the semigroup expansion method is also discussed.

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1. Introduction

There has been a growing interest in exploring non-relativistic (NR) gravity theories [1–28]. In three spacetime dimensions, gravity models can be formulated using the Chern-Simons (CS) formalism [29–31] offering a simpler framework to construct non-relativistic gravity actions. Furthermore, three-dimensional CS gravity theories can be seen as interesting toy models to approach higher-dimensional theories.

The construction of a proper finite NR CS action without degeneracy may require to enlarge the field content of the relativistic theory [32–34]. In the case of three-dimensional Einstein gravity without cosmological constant, it is necessary to consider two additional $U(1)$ gauge fields in order to define a consistent NR limit leading to the so-called extended Bargmann gravity [35,36]. The incorporation of a cosmological constant modifies the theory to the so-called extended Newton-Hooke gravity [37–43]. More recently, a NR version of a three-dimensional gravity theory coupled to electromagnetism has been presented in [44] describing what they called as Maxwellian extended Bargmann (MEB) gravity. Such NR theory requires to introduce three extra $U(1)$ gauge fields to the Maxwell algebra.

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The Maxwell algebra has been introduced in [45–47] in order to describe a Minkowski space in the presence of a electromagnetic field background. In three spacetime dimensions, a CS gravity action without cosmological constant invariant under the Maxwell algebra has been presented in [48–50] whose general solution and asymptotic structure have been studied in [51]. More recently, an isomorphic (dual) version of the Maxwell algebra denoted as Hietarinta-Maxwell algebra has been of particular interest for exploring spontaneous breaking of symmetry [52,53].

A generalization of the Maxwell algebra has been introduced in [54,55] and has been denoted as \mathfrak{B}_5 algebra. This generalization is characterized by the presence of an additional generator with respect to the Maxwell algebra. Interestingly, the aforesaid algebra belongs to a larger family of algebras denoted as \mathfrak{B}_k where \mathfrak{B}_4 and \mathfrak{B}_3 are the Maxwell and Poincaré algebras, respectively. Such family has been useful to recover standard General Relativity without cosmological constant from a CS and Born-Infeld gravity theory [55–58]. Subsequently, the coupling of spin-3 gauge field to \mathfrak{B}_k CS gravity models in three spacetime dimensions has been explored in [59].

It is natural to address the question whether such generalized Maxwell algebra admits a well-defined NR version in three spacetime dimensions. Here we show that the relativistic theory has to be enlarged with four $U(1)$ gauge fields in order to apply an Inönü-Wigner (IW) contraction [60,61] leading to a non-degenerate and finite NR CS gravity theory. The new symmetry obtained corresponds to a generalization of the MEB algebra and has been called GMEB algebra.

An alternative way to find the GMEB symmetry is also discussed considering the semigroup expansion method (S -expansion) [62–66] and following the procedure used in [67]. As was shown in [67], a generalized family of NR algebras, that we have denoted as generalized extended Bargmann algebra, can be obtained using the S -expansion procedure. In particular, we show that the extended Bargmann, the MEB and the GMEB algebras are particular sub-cases of this family of NR algebras. The expansion procedure considered here can be seen as a general method allowing to classify diverse NR symmetries by providing the proper NR limit and the additional gauge fields required in the relativistic theory. Interestingly, the S -expansion method provides not only with the commutation relations of the new NR algebras but also with the non-vanishing components of the invariant tensors which are essential to the construction of NR CS actions.

The paper is organized as follows: in Section 2, we give a brief review of the generalized Maxwell algebra. The corresponding relativistic CS action and its $U(1)$ enlargement are also presented. Sections 3 and 4 contain our main results. In particular, in Section 3 we present the contraction process leading to the GMEB gravity theory. The family of NR algebras obtained through the semigroup expansion procedure is presented in section 4. Section 5 concludes our work with some discussion about possible future developments.

2. Relativistic gravity and generalized Maxwell algebra

In this section we briefly review the generalized Maxwell algebra and present the construction of a three-dimensional CS gravity action invariant under such algebra.

A generalization of the Maxwell algebra has been first introduced as the \mathfrak{B}_5 algebra in [54,55]. It is characterized by the presence of the spacetime rotations J_A , the spacetime translations P_A , the so-called Maxwell gravitational generator Z_A and a new type of generator that we have denoted as N_A . The generators of the generalized Maxwell algebra satisfy the following non-vanishing commutation relations:

$$\begin{aligned} [J_A, J_B] &= \epsilon_{ABC} J^C, & [P_A, P_B] &= \epsilon_{ABC} Z^C, \\ [J_A, P_B] &= \epsilon_{ABC} P^C, & [J_A, N_B] &= \epsilon_{ABC} N^C, \\ [J_A, Z_B] &= \epsilon_{ABC} Z^C, & [Z_A, P_B] &= \epsilon_{ABC} N^C, \end{aligned} \quad (2.1)$$

where $A, B, C = 0, 1, 2$ are the Lorentz indices which are raised and lowered with the Minkowski metric. Here ϵ_{ABC} corresponds to the Levi Civita tensor which satisfies $\epsilon_{012} = -\epsilon^{012} = 1$. It is interesting to point out that the commutator $[P_a, P_b]$ is proportional to the Maxwell gravitational generator Z_A as in the Maxwell algebra. Nevertheless, the commutator $[Z_A, P_B]$ is no longer zero due to the presence of the new generator N_A . Furthermore, unlike the AdS-Lorentz algebra [68–70], this generalization is not a deformation of the Maxwell algebra and then does not reproduce the Maxwell symmetry through a contraction process.

Although this algebra and its generalizations have been explored with diverse applications, a three-dimensional CS gravity action based on this generalization of the Maxwell algebra has not been explicitly presented. A CS action in three spacetime dimensions reads

$$I[A] = \int \langle AdA + \frac{2}{3} A^3 \rangle, \quad (2.2)$$

where $\langle \dots \rangle$ denotes the invariant trace and $A = A^a T_a$ corresponds to the gauge connection one-form. In our case, the connection one-form A is given by

$$A = W^A J_A + E^A P_A + K^A Z_A + U^A N_A, \quad (2.3)$$

where W^A is the spin connection one-form, E^A is the vielbein, K^A is the so-called gravitational Maxwell gauge field and U^A is the new gauge field along the Abelian generator N_A . The respective curvature two form $F = dA + \frac{1}{2}[A, A]$ reads

$$F = R^A(W) J_A + R^A(E) P_A + R^A(K) Z_A + R^A(U) N_A, \quad (2.4)$$

where the Lorentz curvature $R^A(W)$, the torsion $R^A(E)$ and the curvatures along the generators Z_A and N_A are respectively given by

$$\begin{aligned} R^A(W) &:= dW^A - \frac{1}{2} \epsilon^{ABC} W_B W_C, \\ R^A(E) &:= D_W E^A, \\ R^A(K) &:= D_W K^A - \frac{1}{2} \epsilon^{ABC} E_B E_C, \\ R^A(U) &:= D_W U^A - \epsilon^{ABC} K_B E_C. \end{aligned} \quad (2.5)$$

Here $D_W \Theta^A := d\Theta^A - \epsilon^{ABC} W_B \Theta_C$ is the usual Lorentz covariant derivative.

In order to construct the relativistic CS gravity action invariant under the algebra (2.1) we shall consider the most general non-vanishing components of the invariant tensor [55]

$$\begin{aligned} \langle J_A J_B \rangle &= \alpha_0 \eta_{AB}, & \langle P_A P_B \rangle &= \alpha_2 \eta_{AB}, \\ \langle J_A P_B \rangle &= \alpha_1 \eta_{AB}, & \langle J_A N_B \rangle &= \alpha_3 \eta_{AB}, \\ \langle J_A Z_B \rangle &= \alpha_2 \eta_{AB}, & \langle Z_A P_B \rangle &= \alpha_3 \eta_{AB}, \end{aligned} \quad (2.6)$$

where $\alpha_0, \alpha_1, \alpha_2$ and α_3 are arbitrary constants. Then, considering the gauge connection one-form (2.3) and the invariant tensor (2.6) in the general form of a CS action (2.2), one gets

$$\begin{aligned} I_R = \int \left[\alpha_0 \left(W^A dW_A + \frac{1}{3} \epsilon_{ABC} W^A W^B W^C \right) + 2\alpha_1 E^A R_A(W) \right. \\ \left. + \alpha_2 \left(2K^A R_A(W) + E^A R_A(E) \right) \right] \end{aligned}$$