Eur. Phys. J. C (2020) 80:967
https://doi.org/10.1140/epjc/s10052-020-08537-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Asymptotic symmetries of Maxwell Chern–Simons gravity with torsion

H. Adami[1,a], **P. Concha**[2,b], **E. Rodríguez**[3,c], **H. R. Safari**[1,d]

[1] School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran
[2] Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Alonso de Ribera 2850, Concepción, Chile
[3] Departamento de Ciencias, Facultad de Artes Liberales, Universidad Adolfo Ibáñez, Viña del Mar, Chile

Received: 26 June 2020 / Accepted: 10 October 2020 / Published online: 19 October 2020

Abstract We present a three-dimensional Chern–Simons gravity based on a deformation of the Maxwell algebra. This symmetry allows introduction of a non-vanishing torsion to the Maxwell Chern–Simons theory, whose action recovers the Mielke–Baekler model for particular values of the coupling constants. By considering suitable boundary conditions, we show that the asymptotic symmetry is given by the $\widehat{\mathfrak{bms}_3} \oplus \mathfrak{vir}$ algebra with three independent central charges.

Contents

1 Introduction and motivations

Three-dimensional Chern–Simons (CS) gravity theories are considered as interesting toy models since they allow us to approach diverse aspects of the gravitational interaction and underlying laws of quantum gravity. Furthermore, they share many properties with higher-dimensional gravity models which, in general, are more difficult to study. Three dimensional General Relativity (GR) with and without cosmological constant can be described through a CS action based on the AdS and Poincaré algebra, respectively [1–3]. Nowadays, there is a growing interest in exploring bigger symmetries in order to study more interesting and realistic physical models.

Well-known infinite-dimensional enhancements of the AdS and Poincaré symmetries, in three spacetime dimensions, are given respectively by the conformal and the $\mathfrak{bms}_3$ algebras. A central extension of the two-dimensional conformal algebra, which can be written as two copies of the Virasoro algebra, appears as the asymptotic symmetry of three-dimensional GR with negative cosmological constant [4]. In the asymptotically flat case, the three-dimensional version of the Bondi–Metzner–Sachs (BMS) algebra [5–8], denoted as $\mathfrak{bms}_3$, corresponds to the asymptotic symmetry of GR [9]. $\mathfrak{bms}_3$ can be alternatively obtained as a flat limit of the conformal one, in a similar way as the Poincaré symmetry appears as a vanishing cosmological constant limit of AdS. The study of richer boundary dynamics could offer a better understanding of the bulk/boundary duality beyond the AdS/CFT correspondence [10]. Thus, the exploration of new asymptotic symmetries of CS gravity theories based on enlarged global symmetries could be worth studying. In particular, extensions and generalizations of the conformal and the $\mathfrak{bms}_3$ algebras have been subsequently developed in diverse contexts in [11–34].

A particular extension and deformation of the Poincaré algebra is given by the Maxwell algebra. Such symmetry appears to describe a particle moving in a four-dimensional Minkowski background in presence of a constant electromagnetic field [35–37]. This algebra is characterized by the non-vanishing commutator of the four-momentum generator

[a] e-mail: hamed.adami@ipm.ir
[b] e-mail: patrick.concha@ucsc.cl (corresponding author)
[c] e-mail: evelyn.rodriguez@edu.uai.cl
[d] e-mail: hrsafari@ipm.ir

P_a:

$$[P_a, P_b] = M_{ab}, \tag{1.1}$$

which is proportional to a new Abelian generator M_{ab}. The Maxwell algebra and its generalizations have been useful to recover standard GR without cosmological constant from CS and Born–Infeld gravity theories in a particular limit [38–42]. In three spacetime dimensions, an invariant CS gravity action under Maxwell algebra has been introduced in [43] and different aspects of it has been studied [44–53]. As Poincaré symmetry, the Maxwell symmetry describes a three-dimensional gravity theory whose geometry is Riemannian and locally flat. However, the presence of an additional gauge field in the Maxwell case leads to new effects compared to GR. In particular, in [29], the authors have shown that the Maxwellian gravitational gauge field modifies not only the vacuum energy and angular momentum of the stationary configuration but also the asymptotic structure.

To accommodate a non-vanishing torsion to the Maxwell CS gravity theory it is necessary to deform the Maxwell algebra. Here, we show that a particular deformation of the Maxwell symmetry, which we refer to as "deformed Maxwell algebra", allows us to introduce not only a torsional but also a cosmological constant term along the Einstein–Hilbert term. Then, motivated by the recent results on the Maxwell algebra, we explore the effects of deforming the Maxwell symmetry both to the bulk and boundary dynamics. At the bulk level, we show that the invariant CS gravity action under the deformed Maxwell algebra reproduces the Maxwell field equations but with a non-vanishing torsion describing a Riemann–Cartan geometry. Interestingly, the CS action can be seen as a Maxwell version of a particular case of the Mielke–Baekler (MB) gravity theory [54] which describes a three-dimensional gravity model in presence of non-vanishing torsion. Further studies of the MB gravity have been subsequently developed in [55–65]. Here we explore the effects of having a non-vanishing torsion in Maxwell CS gravity at the level of the boundary dynamics. In particular, by considering suitable boundary conditions, we show that the asymptotic symmetry can be written as the $\widehat{\mathfrak{bms}}_3 \oplus \mathfrak{vir}$ algebra. This infinite-dimensional symmetry was recently obtained as a deformation of the infinite-dimensional enhancement of the Maxwell algebra, denoted as $\mathfrak{Max}_3$ algebra [66]. We also show that the vanishing cosmological constant limit $\ell \to \infty$ can be applied not only at the CS gravity theory level but also at the asymptotic algebra, leading to the Maxwell CS gravity and its respective asymptotic symmetry previously introduced in [29].

The paper is organized as follows. In Sect. 2, we present the three-dimensional CS gravity theory which is invariant under a particular deformation of the Maxwell algebra. Furthermore, considering asymptotically flat geometries with null boundary, we discuss the BMS-like solution of the theory. We provide boundary conditions allowing a well-defined action principle. In Sect. 3, we show that the asymptotic symmetry algebra for the Maxwell CS gravity with torsion is given by an infinite enhancement of a deformed Maxwell algebra, which can be written as the direct sum $\widehat{\mathfrak{bms}}_3 \oplus \mathfrak{vir}$. Finally, in Sect. 4 we discuss the obtained results and possible future developments.

Notation We adopt the same notation as [30,66,67] for the algebras; for algebras we generically use "mathfrak" fonts, like $\mathfrak{vir}$, $\mathfrak{bms}_3$ and $\mathfrak{Max}_3$. The centrally extended version of an algebra $\mathfrak{g}$ will be denoted by $\hat{\mathfrak{g}}$, e.g. Virasoro algebra $\mathfrak{vir} = \widehat{\mathfrak{witt}}$.

2 Maxwell Chern–Simons gravity theory with torsion

Using the CS formalism, we present the three-dimensional gravity theory based on a particular deformation of the Maxwell algebra. Unlike the Maxwell case, such deformation leads to a non-vanishing torsion as equation of motion. The deformed Maxwell algebra is spanned by the generators $\{J_a, P_a, M_a\}$, which satisfy the following non-vanishing commutation relations:

$$\begin{aligned}
[J_a, J_b] &= \epsilon_{ab}{}^{c} J_c, \\
[J_a, P_b] &= \epsilon_{ab}{}^{c} P_c, \\
[J_a, M_b] &= \epsilon_{ab}{}^{c} M_c, \\
[P_a, P_b] &= \epsilon_{ab}{}^{c} \left(M_c + \frac{1}{\ell} P_c \right),
\end{aligned} \tag{2.1}$$

where ϵ_{abc} is the three-dimensional Levi-Civita tensor and $a, b = 0, 1, 2$ are the Lorentz indices which are lowered and raised with the Minkowski metric η_{ab}. The ℓ parameter appearing in the last commutator is related to the cosmological constant Λ. Then, the vanishing cosmological constant limit $\ell \to \infty$ reproduces the Maxwell symmetry. Let us note that the Hietarinta–Maxwell algebra [48,52,68] is recovered in the limit $\ell \to \infty$ when the role of the P_a and M_a generators is interchanged. One can see that J_a and P_a are not the generators of a Poincaré subalgebra. However, as it is pointed out in [66], (2.1) can be rewritten as the $\mathfrak{iso}(2,1) \oplus \mathfrak{so}(2,1)$ algebra. This can be seen by a redefinition of the generators,

$$\begin{aligned}
L_a &\equiv J_a - \ell P_a - \ell^2 M_a, \\
S_a &\equiv \ell P_a + \ell^2 M_a, \\
T_a &\equiv -\ell M_a,
\end{aligned} \tag{2.2}$$

where L_a and T_a are the respective generators of the $\mathfrak{iso}(2,1)$ algebra, while S_a is a $\mathfrak{so}(2,1)$ generator. Then, the Lie algebra