A Proximal Solution for a Class of Extended Minimax Location Problem

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Abstract. We propose a proximal approach for solving a wide class of minimax location problems which in particular contains the round trip location problem. We show that a suitable reformulation of the problem allows to construct a Fenchel duality scheme the primal-dual optimality conditions of which can be solved by a proximal algorithm. This approach permits to solve problems for which distances are measured by mixed norms or gauges and to handle a large variety of convex constraints. Several numerical results are presented.

Keywords: Continuous location, minimax location, round-trip location problem, proximal method, Fenchel duality, partial inverse method.

1 Introduction

The aim of this paper is to propose a proximal approach for solving an important class of minimax continuous location problems which in particular contains the round trip location problem [1] and the weighted extended minimax location problem with set up costs [2]. The round trip location problem consists in finding the location of a new facility so that the maximum weighted round trip distance between the new facility and \(n\) pairs of existing facilities (or demand points) is minimized. We mean by round trip distance the total distance travelled starting from the new facility via a pair of existing facilities and going back to the new facility. As example, A.W. Chan and D.W. Hear consider the location of a delivery service. Customers have goods to be delivered from warehouses to retail stores and the objective is to minimize the maximum delivery time.

The extended minimax location problem considered by Drezner [2] is a generalization of the single facility minimax location problem. We want to locate two new facilities such that the maximum trip distance via \(n\) fixed existing facilities (or demand points) is minimized. Here we mean by trip distance the total distance travelled starting from the first new facility via a demand point and going back to the second new facility. As suggested application Drezner considers the
problem of locating emergency hospital services. The total time for dispatching an ambulance and bringing the patient to the hospital consists of the travel time of the ambulance, some setup cost time and travelling back to the hospital. The problem is to shorten the response time for the farthest customer. Observing that placing the ambulance service on the hospital may not be optimal Drezner proposes to consider a priori different sites for the hospital and the ambulance station. Since the time the ambulance takes to get to the patient and the time it takes to bring the patient to the hospital have not the same importance Drezner also suggests different measures of distance for outward journey and journey back. That justifies the use of different norms or gauges.

In our model we are faced to new facilities the locations of which should be optimally determined via a min-max criterion. It is well known that min-max criteria induce nondifferentiability in optimization problems. In the framework of continuous location analysis, this nondifferentiability is also due to the fact that a norm is never differentiable at the origin and to the (possible) use of polyhedral norms such as the $\ell_1$ norm, the Tchebychev norm, etc. The nondifferentiability prevents the use of standard optimization methods and leads to solving problems by adapted procedures. Algorithms based on linear programming have already been developed for round trip problems involving the rectilinear norm [4]. Drezner proposed to solve unconstrained round trip location problems by a trajectory approach. Other procedures like those explained in [3] have also been studied. All these approaches are not completely satisfactory in particular because they often cannot be extended to problems involving mixed norm and/or non linear constraints. The aim of this paper is to investigate the interest of considering proximal procedures, like those developed in [5], and which exploit in depth the very special structure of minimax location problems.

A major technical difficulty which frequently arises when one wants to implement proximal algorithms is the computation of the proximal iteration. We will show that this difficulty can be overcome by working on an equivalent decomposable formulation of the problem. The idea is to incorporate the non linear constraints in the objective function via penalization terms, to split the objective function in independent terms and to handle by duality all the original linear constraints and the linear relations induced by the splitting. The role of the splitting is to decompose the objective function as a sum of independent terms in such a way the proximal mapping associated to the subdifferential of each term can be effectively and easily computed. This can be done by several tricks as duplication of some variables and alternative representation of certain convex functions. The original linear constraints as well as the linear relations induced by the splitting are conserved as constraints because they can be easily treated by Fenchel duality. Another fact which often militates against the use of proximal procedures is the slow rate of convergence. This second difficulty cannot be completely eliminated. However, as already observed in [5] proximal procedures seem to perform rather well on location problems and their efficiency can be significantly improved by a judicious scaling on the data. These procedures have also important advantages, as robustness and stability. Their use allows a great flexibility in using mixed norms and different types of (convex) constraints.