

# UNFOLDING OF THE HAMILTONIAN TRIANGLE VECTOR FIELD

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**Abstract.** In this paper we study unfoldings of the Hamiltonian triangle vector field within quadratic vector fields. We complete and correct some previous results of Zoladek [18].

## 1. Introduction and main result

In this paper we study unfoldings of the quadratic hamiltonian triangle vector field within the quadratic vector fields. These unfoldings have been studied by Zoladek [17], [18] and Iliev [6] [7] among others. Our initial motivation was to study this example as the first multi-parameter example where iterated integrals cf. Gavrilov [4] appear in the principal part of the displacement function. Linear one parameter families of vector fields unfolding Hamiltonian products of lines have been extensively studied by Uribe in [13] and [14]. Having difficulty to read the paper [18], we did ourself the calculations. It turned out that the displacement function is not precisely of the form given by Zoladek. We agree with the ideal of coefficients in which the displacement function divides, but one of the principal parts  $J_4$  is essentially different. It is an integral of a rational function and cannot be written as an integral of a polynomial function as can be seen from the fact that its integral does not vanish together with the vanishing cycle at the center.

In [7] Iliev studies the principal parts  $J_1$ ,  $J_2$ ,  $J_3$  and  $J_4$  of the displacement function as obtained by Zoladek. He shows that these functions form a Chebyshev system. In fact, due to the error in Zoladek's paper the study of the Chebyshev property of the true principal part functions as given in the present paper remains to be done.

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