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THE PERIOD FUNCTION IN A CLASS OF QUADRATIC KOLMOGOROFF SYSTEMS *

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Abstract

In this paper we consider the family of quadratic Kolmogoroff systems with a center in the real quadrant:

$$\begin{cases} \dot{x} = x(1 - x - ay) \\ \dot{y} = y(-1 + ax + y) \end{cases},$$

where $1 < a < \infty$. This system has three invariant lines (the coordinate axes and the line $x + y - 1 = 0$) and a family of periodic solutions nested around a center and filling out the triangle determined by the three invariant lines. Using integrability of this system we reduce the abelian integral representing the period function and its derivative. The main result is that the corresponding period function is monotone increasing for values of the parameter near $a = 3$.

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1. Introduction

A Kolmogoroff system is given by a planar differential system of the form

$$\begin{cases} \dot{x} &= xF(x,y) \\ \dot{y} &= yG(x,y) \end{cases}$$

with F, G of C^1 class.

This kind of system appears usually in predator-prey models and so only dynamics on the first quadrant is considered. In this paper we consider the quadratic Kolmogoroff system:

$$\begin{cases} \dot{x} &= x(1-x-ay) \\ \dot{y} &= y(-1+ax+y) \end{cases} , \tag{1.1}$$

where $1 < a < \infty$.

A system of this class has a family of periodic solutions nested around a center at $\left(\frac{1}{1+a}, \frac{1}{1+a}\right)$ and filling out the triangle formed by the saddle points $(0,0), (1,0)$ and $(0,1)$. The family of periodic solutions forms a center period annulus

The function which associates to any closed curve its period, is called the period function. We are interested in obtaining the global description of the period function defined on the center period annulus.

In section 2 , we analyze the period function and we prove that it is monotone increasing for $a = 3$.

In section 3 we prove, by means of a linear change of coordinates that system (1.1), in the Bautin's form is of type B_2 for all $a > 1$ Bautin's systems and their period functions were defined in [1] by C. Chicone and M. Jacob where they conjectured that for B_2 systems the period function is globally monotone increasing.

By calculating the first three periodic coefficients for $a > 1$, we realize that they are alternatively positive and negative. This result suggests us that there would exists a value of the parameter a such that the corresponding system has a period function with critical points.

In general, aside from their intrinsic interest, monotonicity properties of the period function are important in the question of existence