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GAIN OF REGULARITY FOR AN NONLINEAR DISPERSIVE EQUATION KORTEWEG - DE VRIES - BURGERS TYPE *

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Abstract

In this papers we study smoothness properties of solutions. We consider the equation of Korteweg - de Vries - Burgers type

$$(1) \qquad \begin{cases} u_t + \partial_x f(u) = \epsilon\, \partial_x^2 u - \delta\, \partial_x^3 u \\ u(x,\, 0) = \varphi(x) \end{cases}$$

with $-\infty < x < +\infty$ *and* $t > 0$. *The flux* $f = f(u)$ *is a given smooth function satisfying certain assumptions to be listed shortly. It is shown under certain additional conditions on* f *that* C^∞ *- solutions* $u(x,\, t)$ *are obtained for all* $t > 0$ *if the initial data* $u(x,\, 0) = \varphi(x)$ *decays faster than polinomially on* $\mathrm{IR}^+ = \{\, x \in \mathrm{IR}\, ;\ x > 0\,\}$ *and has certain initial Sobolev regularity.*

Keywords and Phrases *: Evolution equations, Lions - Aubin Theorem, Weighted Sobolev Space.*

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1. Introduction

In 1976, J. C. Saut and R. Temam [22] have remarked that a solution u of an equation of Korteweg-de Vries type cannot gain or lose regularity: They show that if $u(x, 0) = \varphi(x) \in H^s(\mathrm{IR})$ for $s \geq 2$, then $u(\cdot, t) \in H^s(\mathrm{IR})$ for all $t > 0$. The same results were obtained independently by J. Bona and R. Scott [2] by different methods. For the Korteweg - de Vries (KdV) equation on the line, T. Kato [16], motivated by work of A. Cohen [6], showed that if $u(x, 0) = \varphi(x) \in L^2_b \equiv H^2(\mathrm{IR}) \bigcap L^2(e^{bx}\, dx)$ $(b > 0)$ then the solution $u(x, t)$ of the KdV equation becomes C^∞ for all $t > 0$. A main ingredient in the proof was the fact that formally the semi-group $S(t) = e^{-t\,\partial_x^3}$ in L^2_b is equivalent to $S_b(t) = e^{-t\,(\partial_x - b)^3}$ in L^2 when $t > 0$. One would be inclined to believe this was a special property of the KdV equation. This is not, however, the case. The effect is due to the dispersive nature of the linear part of the equation. S. N. Kruzkov and A. V. Faminskii [20] for $u(x, 0) = \varphi(x) \in L^2$ such that $x^\alpha \varphi(x) \in L^2((0, +\infty))$ is was proven that the weak solution of the KdV equation constructed there has l-continuous space derivatives for all $t > 0$ if $l < 2\alpha$. The proof of this result is based on the asymptotic behavior of the Airy function and its derivatives, and on the smoothing effect of the KdV equation found in [16, 20]. Corresponding work for some special nonlinear Schrödinger equations was done by Hayashi et al. [12, 13] and G. Ponce [21]. While the proof of T. Kato appears to depend on special a priori estimates, some of its mystery has been resolved by results of local gain of finite regularity for various others linear and nonlinear dispersive equations due to P. Constantin and J. C. Saut [10], P. Sjolin [23], J. Ginibre and G. Velo [11] and others. However, all of them require growth conditions on the nonlinear term.

All the physically significant dispersive equations and systems known to us have linear parts displaying this local smoothing property. To mention only a few, the KdV, Benjamin-Ono, intermediate long wave, various Boussinesq, and Schrödinger equations are included. Continuing with the idea of W. Craig, T. Kappeler and W. Strauss [9] we study a equation of Korteweg - de Vries - Burgers Type

$$u_t + \partial_x f(u) = \epsilon\, \partial_x^2 u - \delta\, \partial_x^3 u \tag{1.1}$$